On the Metric Dimension of Extended de Bruijn and Extended Kautz Digraphs

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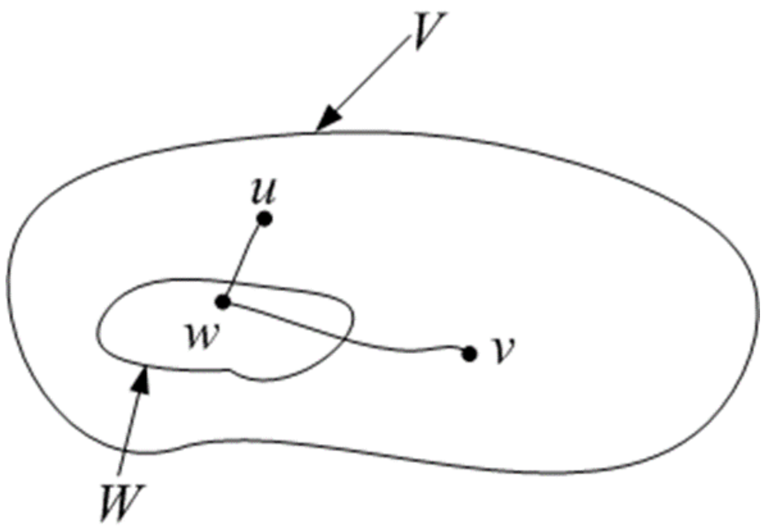
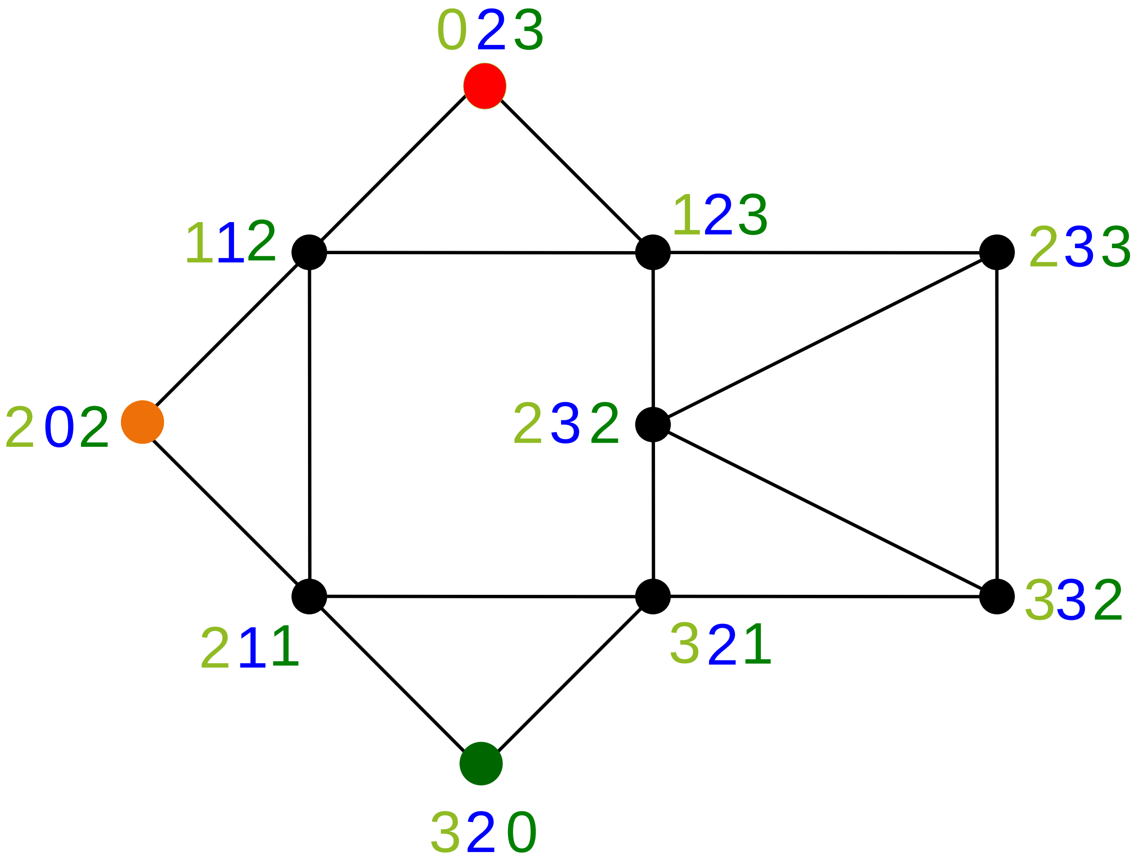
Metric Basis

- A metric basis for a digraph $G(V, A)$ is a minimum set $W \subset V$ such that for each pair of vertices u and v of V, there is a vertex $w \in W$ such that the length of a shortest directed path from w to u is different from the length of a shortest directed path from w to v in G; that is $d(w, u) \neq d(w, v)$.

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- Metric Basis
- Resolving set
- Locating set
- Reference set

- The concept of a metric basis has appeared in the literature under different names as early as 1975.
- Slater called these sets ***locating sets*** and ***reference sets*.**
- Independently, Harary and Melter discovered these concepts as well but used the terms ***metric basis*** and ***metric dimension***, rather than reference set and location number.
- Chartrand et al. used the terms ***resolving set*** and ***minimum resolving set*.**

Previous Results

- A connected graph G of order n has dimension **1** if and only if $G = P_n$.
- A connected graph G of order $n > 2$ has dimension $\boldsymbol{n-1}$ if and only if $G = K_n$.
- Let G be a connected graph of order $n > 4$. Then $\beta(G) = \boldsymbol{n-2}$ if and only if $G = K_{s,t}$; $G = K_s + K_t'$; or $G = K_s + (K_1 \, U \, K_t)$.

{Chartrand et al, 2000}

Let $G = (V, E)$ be a graph with metric dimension 2 and let $\{a, b\}$ be a metric basis in G. The following are true.

1. There is a unique shortest path P between a and b.
2. The degrees of a and b are at most 3.
3. Every other node on P has degree at most 5.

The De Bruijn graph $B(d, n)$

- A De Bruijn graph $B(d, n)$ is a directed graph on d^n vertices and d^{n+1} directed edges.
- $V = \{x_1, x_2, \dots x_n : x_i \in \{0, 1 \dots d-1\},\ i = 1,2 \dots n\}$.
- There is an edge from x to y, if and only if $x = x_1, x_2, \dots x_n$, and $y = x_2 \dots x_n \alpha$, where $\alpha \in \{0, 1 \ \dots\ d-1\}$.

- The De Bruijn graph is regular, Eulerian and Hamiltonian. It has small diameter, nearly optimal connectivity, simple recursive structure, simple routing algorithm and contains some other usual topologies as its subgraphs.

- The metric dimension of $B(d, n)$ is given as
- $\beta\left(B(d,n)\right) = d^{n-1}(d-1)$

{Bharati et al, 2000}

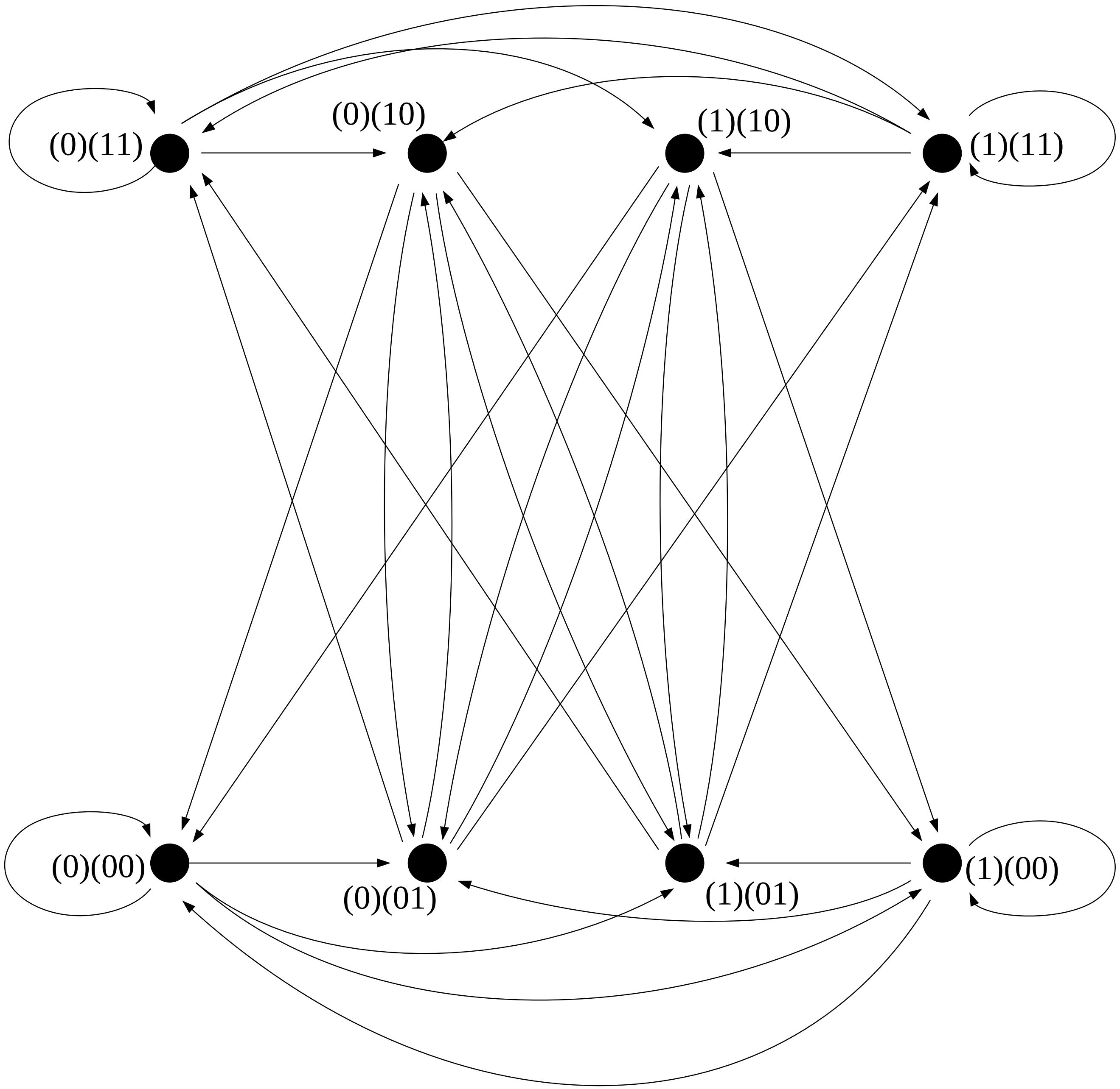
The extended de Bruijn graph

- The extended de Bruijn graph $EB(d,m;\ q_1,\ q_2,\dots q_p)$ is defined as follows
- The set of vertices is the set of m-dimensional vectors on d elements divided into p blocks of sizes $(q_1,\ q_2,\dots q_p)$ written in the form
- $(x_{11},\ x_{12},\dots x_{1q_1})(x_{21},\ x_{22},\dots x_{2q_2})\dots(x_{p1},\ x_{p2},\dots x_{pq_p})$

where $0 \leq x_{ij} \leq\ d-1$, and $\sum_{i=1}^{p} q_i = m$

- . A vertex of is joined by an arcs to all vertices of the form

$(x_{12},\dots x_{1q_1},\alpha_1)(x_{22},\dots x_{2q_2},\alpha_2)\dots(x_{p2},\dots x_{pq_p},\ \alpha_p)$ where $\alpha_i,(0 \leq i \leq\ d-1)$ is taken over $\{0,1,\dots d-1\}$.



EB(2,3;1,2)

- Let G be a directed graph and $C = \{c_1, c_1, \ldots c_k\}$ be a subset of the vertex set V such that $N^{-}(c_1) - C = N^{-}(c_2) - C = \cdots = N^{-}(c_k) - C = N^{-}(C) - C$. Then all elements of C but one should be present in any metric basis of G.

- $\beta\left(EB(d, m;\ q_1, q_2, \dots q_p)\right) = d^{m-p}(d^p - 1).$

- $\beta\left(E_k\left(d,m;\ q_1,\ q_2,\ldots q_p\right)\right)=\frac{d^{m-p}(d+1)^p}{d^p}\left(d^p-1\right).$

Thank You